

ACEX2022

15th International Conference on Advanced Computational Engineering and Experimenting,
Florence, from 3-7 July 2022

On the Uniqueness of the Solution and the Variational Principles Within the Linear Coupled Strain Gradient Elasticity

Strain gradient elasticity, in that the strain energy density is a function of the strain and the second gradient of the displacement vector, is a natural extension of the classical theory of elasticity and is a particular case of higher-order gradient material theories. It has a long history. Since the beginning of the last century, in order to avoid the shortcomings of the classical theory of elasticity, a variety of non-classical theories have been proposed.

In gradient elasticity, the proofs of fundamental theorems are presented mainly for the uncoupled cases. The existence and uniqueness theorem is the crux of any theory. The principle of the minimum of the elastic potential, which is minimized over the displacement field, and the principle of the minimum of the complementary potential, which is minimized over the stress field, are elementary variational formulations of the elastostatic boundary value problem. They serve as starting point for the construction of approximate solutions, they are needed for different mean value theorems and they give upper bounds for the stored elastic energy and the global stiffness of elastic systems. All of this has important practical applications, for numerical solution methods like the FEM (approximate solutions) and as bounds for effective stiffness's in homogenization theory (upper bounds). Likewise, maximum principles have been formulated. In homogenization theory, they give with the absence of body forces the lower bounds for the stored energy and hence for the effective stiffness.

In this presentation, these fundamental theorems will be proved and discussed within the linear coupled strain elasticity theory.